TOPOLOGY AND SHAPE OPTIMIZATION OF SHEET METALS WITH INTEGRATED DEEP-DRAWING-SIMULATION

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Keywords: Topology optimization, Sheet Metals, Deep Drawing, Manufacturing Simulation, Manufacturing Constraint.

Abstract. Shell structures manufactured by deep drawing are important components of lightweight designs for industrial purposes. Shape optimization is excessively used in order to improve the shells performance. Thereby the mid surface design can be changed, but the topology is fixed. Topology optimization can be used to introduce cut-outs into shell structures, but then the mid surface design is fixed. Furthermore there has been no attempt to consider the deep drawing process as manufacturing simulation in the optimization process.
In order to optimize the mid surface design and cut-outs simultaneously, a new topology optimization approach for shell structures has been developed [1]. The current paper focuses on the implementation of the nonlinear deep drawing simulation to the topology optimization based on linear finite element calculations. It is checked if tearing is likely to occur during the deep drawing process and according to the deep drawing results, the shell metal structure is smoothed locally at critical locations to avoid the tearing.

1 INTRODUCTION

Topology optimization is already widely used in order to get ideas for lightweight constructions. Without manufacturing constraints the topology optimization results can usually only be manufactured by 3D-printing because of the neglect of the manufacturing process and the complex resulting geometries with an arbitrary change of the cross sections and undercuts.
For several production processes some geometrical requirements on the final design can be handled during the topology optimization. For example casting parts should not have undercuts in the direction of demolding and a draft angle. Sheet metals manufactured by deep drawing should have a constant wall thickness and no undercuts in punch direction [1].
But fulfilling these geometrical requirements are not sufficient to guarantee a successful manufacturing. The casting mold can possibly not be completely filled or solidifies with porosities or shrinking defects. The deep drawing of sheet metals can possibly initiate cracks (tearing) in the blank (e.g. at low corner radii) or result in wrinkling (e.g. because of low blank holder force).
This paper focuses on the prevention of tearing at low corner radii during the topology optimization of deep drawn sheet metals. This defect is caused by the geometry of the sheet metal and can therefore be prevented by a slight modification of the structure. The process parameter (blank holder force, friction etc.) are considered as being constant and no design variables. This parameter of the deep drawing process can be used as tuning factors to prevent other defects by maintaining the blank’s geometry.
2 TOPOLOGY OPTIMIZATION APPROACH

As topology optimization algorithm the artificial element densities are updated gradient based using the SIMP approach [2] [3]

\[ E_i = x_i^s E_0 \quad x_i \in [0, 1], \]

where \( x_i \) are the artificial element densities with \( x_i \approx 0 \) representing void and \( x_i \approx 1 \) representing solid material. \( E_i \) is the Young’s Modulus of element \( i \) and \( E_0 \) is the Young’s Modulus of the solid material. An increasing penalty exponent \( s > 1 \) results in a lower stiffness of intermediate density elements. The design space is discretized with 8-node trilinear finite elements in a structured grid (voxel elements).

The global stiffness matrix \( K \) is assembled from the element stiffness matrices \( K_{E_i} \)

\[ K = \sum_i x_i^s A_i^T K_{E_i} A_i \]

with \( A_i \) being the assembly matrices, which assign the local degrees of freedom (DOFs) of each element to the global DOFs \( u_{E_i} = A_i u \). \( u \) is the global displacement vector, \( u_{E_i} \) is an element displacement vector. In order to calculate the displacements \( u \) from the loads \( f \) in a linear-static finite element calculation

\[ Ku = f, \]

the global stiffness matrix \( K \) has to be non-singular, which leads to a lower bound \( x_i \geq x_{\text{min}} \). In this paper \( x_{\text{min}} = 0.001 \) is chosen.

The mathematical formulation of the optimization problem reads as follows

\[ \begin{align*}
\min_x & \quad c(x) = \frac{1}{2} u^T Ku \\
\text{Subject to} & \quad v_f(x) - v_f^{\text{max}} = \sum_i x_i - v_f^{\text{max}} \leq 0 \\
& \quad Ku = f \\
& \quad x_{\text{min}} \leq x \leq 1
\end{align*} \]

with the mean compliance \( c \) as objective function and the constrained volume fraction \( v_f \). The number of design elements is \( n_d \). Non-design Elements have an element density of \( x_i = 1 \).

The sensitivities can be calculated analytically (assuming non-variable loads, e.g. no gravity)

\[ \frac{dc}{dx_i} = -\frac{1}{2} x_i^{s-1} u_{E_i}^T K_{E_i} u_{E_i} \]

\[ \frac{dv_f}{dx_i} = \frac{1}{n_d}. \]
To ensure an existence of the solution to the topology optimization problem and in order to prevent checkerboarding, the sensitivities are filtered and thereby the local variation of the density field is restricted [4]. The sensitivities are smoothed by calculating a weighted average of the sensitivities from the neighbor elements

$$\frac{de}{dx_i} = \sum_j w_j x_j \frac{de}{dx_j} \quad \text{with} \quad w_j = \max(r - \text{dist}(i,j), 0)$$

(6)

with the weights $w_j$ depending on the filter radius $r$ and distance $\text{dist}(i,j)$ of the mid points of element $i$ and element $j$.

The method presented in this paper also works with other filter techniques like density filtering.

The element densities are updated based on the filtered sensitivities. Therefore the Method of Moving Asymptotes (MMA, [5]) is combined with the dual optimization method [6], which is very efficient, if only a few constraints are considered.

### 3 DEEP DRAWING CONSTRAINT IN TOPOLOGY OPTIMIZATION

In order to obtain an optimized structure without undercuts and with an approximately constant wall thickness, the objective sensitivities are penalized, if they are far away from the mid surface [1]. Therefore the user has to define the desired wall thickness $b$ (unit: element edge lengths) and the punch direction.

![Figure 1: Calculation of mid surface](image)

Figure 1 shows the calculation of the mid surface at an exemplary cross section. The mid surface point per column of width $w$ is calculated by averaging the element position $\xi_i$ with the element densities $x_i$ of all elements, which element midpoint is located within the column

$$\xi_m = \frac{\sum \xi_i \cdot x_i}{\sum x_i}.$$  

(7)

Finally the filtered sensitivities of the objective function are penalized, in order to end up with a thin walled structure, by multiplying with
\[ P_i = \frac{1 + \exp(-f \cdot a)}{1 + \exp(2a / b \cdot (d_i - f))} \qquad f = \frac{\ln\left(-2 + \exp\left(a \cdot \frac{b}{2}\right)\right)}{a}. \]  

\(d_i\) is the shortest distance between element \(i\) and the mid surface, \(a\) is the discreteness of the penalty function.

By artificially downgrading the objective sensitivities far away from the mid surface, the element densities outside the sheet metal decrease and a shell-like structure develops. The mid surface calculated by equation (7) cannot have undercuts in punch direction and can therefore possibly be deep drawn in one step. The penalization of sensitivities far away from the mid surface ensures, that the optimized result is a thin walled structure with a limited thickness. The filter radius \(r\) imposes a minimum length scale, which should be about 0.7 to 0.9 times the desired wall thickness \(b\) in order to end up with a nearly constant wall thickness. The mid surface can move during the optimization by redistributing the element densities to the side of the shell, where the objective sensitivities are smaller. The compliance’s sensitivities are non-positive, which means the shell moves towards the side with the larger absolute values of the objective sensitivities.

### 4 MANUFACTURING PROCESS

Deep drawn sheet metals with cut-outs can be manufactured by different manufacturing steps. Cut-outs can be introduced by punching or laser-cutting. Punching is usually the cheaper process. If the sheet metal is already heat treated during the manufacturing process by a laser or the flat sheet metal blank is laser-cut from the coil, cut-outs can also be introduced by laser-cutting without a great additional effort.

These steps can be performed before or after the deep drawing. Deep drawing with cut-outs is only possible with a limited variety of cut-outs, because cracks are often initiated at those cut-outs and tearing occurs. It is also difficult to meet tolerances of cut-outs in the deep drawn component, if the deep drawing is already done with cut-outs.

Because of this reasons we assume, that the sheet metal is first deep drawn without cut-outs and the cut-outs are laser-cut or punched with cam slide units afterwards.

In order to use the mid surface, derived from equation (7), for a deep drawing simulation, the holes in the mid surface have to be filled. This is done by a linear interpolation of the \(\xi\)-coordinates. The filling of holes in the mid surface is done before applying the following manufacturing constraints and before performing the deep drawing simulation.

### 5 GEOMETRICAL ADJUSTMENT OF THE MID SURFACE

Because the shape of the optimized sheet metal described by voxel elements is governed by the mid surface, a change of the mid-surface results in an adjustment of the voxel structure. This is used in order to implement an additional manufacturing constraint. For deep drawing it is common to use a corner radius

\[ r_c \geq 2b. \]  

This can be reached by controlling the curvature of the mid surface. If this constraint is not fulfilled at one position of the shell, the shell around this position is smoothed locally by moving the mid surface points parallel or antiparallel to the punch direction.

The mid surface, derived from equation (7), can be interpret as shell mesh with the mid surface points being the nodes. Because of segmenting the design space in equidistant columns (see Figure 1) this shell
mesh is structured, i.e. the nodes are equidistant in the both directions perpendicular to the punch direction. For every 4-node shell element the normal vector is calculated. If the angle difference between to neighbor elements exceeds a limit angle, the shell has to be smoothed locally. The limit angle

\[
\alpha_{\text{max}} = 2 \arcsin \left( \frac{1}{r_{\text{c min}}} \right)
\]

is derived from Figure 2 (unit of \( r_{\text{c min}} \): element edge lengths). A semi-circle with radius \( r_c = \frac{r_{\text{c min}}}{2} \) is assumed as a mid surface calculated by equation (7). These circle is discretized using equidistant nodes in the direction \( \zeta \), which is perpendicular to the punch direction \( \xi \), as it comes from the mid surface calculation (see Figure 1). The smallest angle difference between neighbor elements is shown with red arrows, representing the element normal. Their angle difference is \( \alpha_{\text{max}} \).

![Figure 2: left: Discretized semi-circle, right: enlarged section, all units: element edge lengths](image)

If it can be assured, if all angle differences between neighbor elements \( \alpha_i < \alpha_{\text{max}} \), then the manufacturing constraint from equation (9) is fulfilled.

To achieve that the \( \zeta \)-coordinates of the mid surface nodes are filtered

\[
\tilde{\zeta}_{\text{mi}} = \frac{\sum_j \zeta_{mj} \cdot w_j}{\sum_j w_j} \quad w_j = \max(r_{si} - \text{dist}(i,j), 0).
\]

The smoothing filter radius \( r_{si} \) is not constant. It is set to \( r_{si} = 1 \) element edge length (no smoothing), if the angle difference \( \alpha_i \) at this node is already smaller than \( \alpha_{\text{max}} \), in order not to adjust the shell in already smooth areas. If the manufacturing constraint (9) is violated locally, the local filter radius is set to

\[
r_{si} = \frac{\alpha_i}{\alpha_{\text{max}}} + 1 \text{ element edge length}.
\]

If there are discontinuities in the smoothing filter radius \( r_{si} \) between neighbor nodes, the filtered node coordinates can also have discontinuities, which causes a violation of the manufacturing constraint. Thus the smoothing filter radius \( r_{si} \) also have to be filtered.
\[
\tilde{r}_{ij} = \frac{\sum_{j} r_{sj} \cdot w_j}{\sum_{j} w_j}
\]

\[
w_j = \max(r_j - \text{dist}(i,j), 0)
\]

with the constant filter radius \(r_r\).

This smoothing method is shown exemplary in Figure 3 using the cosine function \(\xi = \cos(\zeta)\) discretized with 20 nodes and with \(\alpha_{\text{max}} = 8^\circ\) respectively \(\alpha_{\text{max}} = 4^\circ\) and \(r_r = 2\) element edge lengths. Figure 3a and b shows, that the filtering by weighting the neighbor coordinates is critical at the boundary of the design space, i.e. at the ends of the cross section curve. Usually their neighbor nodes are either all above or all under the \(\xi\)-coordinate at the boundary, thus the ends are smoothed towards lower absolute values of the \(\xi\)-coordinate. Because the position of the boundary nodes is also one outcome of the optimization, this results in a suboptimal translation and distortion of the shell at the boundary, which is not wanted. To resolve this issue, additional artificial mid surface points created outside the boundary (light blue area in Figure 3c), which are symmetric to the boundary node. Thereby the boundary nodes remain at the same position (see Figure 3c).

![Figure 3: Smoothing example](image)

It can happen, that the manufacturing constraint (9) is not fulfilled after the first smoothing. Therefore the suggested smoothing of the mid surface is performed iteratively until \(\alpha_i < \alpha_{\text{max}}\) is fulfilled and thereby all corner radii are smaller than the double wall thickness.

6 LOCAL ADJUSTMENT OF THE MID SURFACE DUE TO TEARING

A similar smoothing procedure is used in order to prevent tearing during the deep drawing process. Therefore the smoothed shell mesh is used for a deep drawing simulation. The commercial software AutoForm OneStep® is used. It uses an inverse approach [7] by an iterative determination of the points in the initial blank corresponding to the points of the final product, which is provided as input data. Beside the mesh of the formed sheet metal, the material data including the stress-strain curve, the initial shell thickness, the friction coefficient and the boundary conditions (restraining at edges) have to be specified. The nonlinear solver assumes a linear strain path and an isotropic material behavior. The sheet metal is treated as membrane with additional bending stiffness [8]. Due to this simplifications the solving usually lasts only a few seconds and the method is straightforward to implement to the optimization, because the tools do not need to be modelled explicitly and therefore the preprocessing can be efficiently handled as
batch process without interaction with the user. However the inverse approach is an idealization of the real
deep drawing process and its results are not as accurate as the results from an incremental forming simu-
lution. The OneStep-results does not allow a precise prediction of wrinkling. But an incremental forming
simulation can be hardly handled as a batch process and takes an excessive calculation time.

As result the principal strains are extracted in order to evaluate the likelihood of tearing. Therefore the
forming limit diagram (FLD) is evaluated. The FLD depends on the material and the wall thickness. An
example for a high-strength complex-phase steel with a thickness of 1 mm is shown in Figure 4. The
important curve for tearing is the forming limit curve (FLC). Above this curve the sheet metal probably
tears. Often a safety factor is applied, in order to avoid the risk of tearing. This zone is highlighted in
yellow and is limited from a curve, which has in this example a vertical offset of 20 % from the FLC.

Another critical area is the excessive thinning shown in orange with an exemplary thinning of >30 %. In
the zone of thickening (purple) the shell probably wrinkles, wrinkling is also possible in the zone of com-
pression (blue).

This work focuses on tearing. The tearing criterion \( c_t \)

\[
c_t = \frac{a_t}{b_t}
\]

is evaluated elementwise (exemplary element results shown as black dot in Figure 4).

The tearing criterion has to fulfil the manufacturing constraint

\[
c_t \leq c_{t,max}.
\]

with \( c_{t,max} \) as an user defined limit (e.g. \( c_{t,max}=0.8 \)).

If the tearing criterion is violated locally \((c_t>c_{t,max})\), the nodes around that element are smoothed with the
local filter radius

\[
r_e = \frac{c_t}{c_{t,max}} + 1 \text{ element edge length}
\]

analogous to the smoothing because of a small corner radius in chapter 5.
Figure 5 shows the FLD of a ductile steel in contrast to the brittle steel in Figure 4. It can be seen that FLC of the ductile steel has a higher position. Thus a higher degree of plastic deformation is possible. Figure 5b shows the FLD of the same ductile steel, but with an increased thickness. Again higher strains are permitted. This does not mean in general, that thicker sheet metals can be cold-formed to a stronger curvature, because at the same curvature the strain of the inner and outer shell fiber is larger considering an increased wall thickness.

7 APPLICATION EXAMPLES

As application example a cantilever beam (see Figure 6) is optimized. The compliance is minimized considering a volume fraction constraint $v_{f_{\text{max}}} = 6.25\%$. The design space is discretized with about 500000 elements with an element edge length of 2.5 mm. The rear surface of the structure is clamped, at the front bottom edge a line load of $q = 200$ N/mm is applied. The elements at the line load are defined as non-design space. A sensitivity filter with the radius of $r = 4.25$ mm and a penalty exponent $s = 3$ are used. The material is steel with Young’s Modulus $E_0 = 210$ GPa and Poisson’s ratio $\nu = 0.3$. The element densities are initialized uniformly in the design domain. The desired wall thickness is $b = 7.5$ mm and negative $z$-direction is chosen as punch direction.

For the deep drawing simulation the friction coefficient is set to $\mu = 0.15$. As restraining value 0.3 is chosen (0 – free, 1 – locked), which is an abstraction of the blank holder forces. Low restraining results in wrinkling, strong restraining can result in tearing [9]. The restraining is applied to the shell edges at $y = 0$ mm and $y = 200$ mm. For the limit on the tearing criterion $c_{\text{tmax}} = 0.8$ is selected. The material properties are shown in Figure 7. The FLC corresponds with the FLD in Figure 4. For the smoothing the filter radius $r_r = 2$ element edge lengths is used.

Figure 6: Application Example Cantilever Beam – Load case and design space
Without the manufacturing constraints from chapter 5 and 6 a compliance of 17.46 Nmm is reached (Figure 8). These results are postprocessed by smoothing the mesh with respect to the minimum corner radius (Figure 9). The smoothing takes 26 iterations with a calculation time less than 1 second. In the result the minimum corner radius is met, but the maximum calculated tearing criterion $c_t=2.37$ shows, that tearing will occur during the seep drawing (Figure 9c).

In the following example the manufacturing constraints with respect to the corner radius and the tearing criterion are both active during the topology optimization. The same design space and load case is used. To show the performance of the smoothing algorithm, the smoothing during the first iteration is presented in Figure 10. Due to the uniform initial density distribution of $x_i=0.0625$, the mid surface is located in the middle of the design space. On the loaded edge non-design elements with $x_i=1.0$ are defined. Therefore the mid surface is located lower and a sharp edge occurs at this position (Figure 10b). To meet the corner radius criterion, the mid surface is smoothed, which takes 10 smoothing iterations.
Figure 10c shows, that a constant corner radius results, whereby the position of the front edge remains at the same (at $\zeta_1=300$ mm: $\xi_i=\bar{\xi}$) because of the process shown in Figure 3c. The tearing criterion restriction is not active in the first iteration.

The optimization results are shown in Figure 11. It can be seen, that the voxel structure adapted to the shape of the mid surface. The compliance of $c=21.28$ Nm is 22 \% worse than in the first example due to the additional manufacturing constraints. The topology and shape of the optimized structure is extensively different from the optimized structure in Figure 8, because the corner radius constraint is active at a lot of locations. The curvature at the clamped end is smaller because of the tearing criterion constraint. The maximum tearing criterion is $c_t=0.77$, which fulfills the constraint $c_t \leq c_{\text{max}}=0.8$.  

**Figure 10**: Smoothing according to minimum corner radius in first iteration

**Figure 11**: Optimization results with manufacturing restriction for corner angle according to equation (9) and tearing criterion according to equation (15)
8 COMPARISON AND CONCLUSION

Manufacturing constraints for minimum corner radius and a maximum tearing risk has been introduced to the topology optimization of deep drawable sheet metals. These manufacturing constraints are realized by smoothing of the mid surface. All considered manufacturing constraints are heuristics, which are non-differentiable, and govern the gradient based topology optimization approach.

For the tearing criterion a deep drawing simulation has been implemented to the optimization. Because of the simplified deep drawing simulation using an OneStep-solver the calculation time of one iteration increases only slightly. Often a larger number of iterations is needed, when additional constraints are considered.

As shown in the application example the performance of the structure decreases due to the additional constraints. Completely different topologies can develop because of the new manufacturing constraints. As benefit the optimized structures are close to a manufacturable design. If the manufacturing constraints would not be considered during the optimization, the structures have to be modified significantly afterwards, resulting in a suboptimal design.

REFERENCES